Table 14.1

Some commonly-used orthogonal polynomials.

name	symbol	interval	weight function
Legendre	$P_n(x)$	$-1 \le x \le 1$	1
Chebyshev(Tchebychef)	$T_n(x)$	$-1 \le x \le 1$	$(1-x^2)^{-1/2}$
Laguerre	$L_n(x)$	$0 \le x < \infty$	e^{-x}
Associated Laguerre	$L_n^{(\alpha)}(x)$	$0 \le x < \infty$	$x^{\alpha}e^{-x}$
Hermite	$H_n(x)$	$-\infty < x < \infty$	e^{-x^2}
Hermite	$He_n(x)$	$-\infty < x < \infty$	$e^{-x^2/2}$

Table 14.2

The recursion formulas of the orthogonal polynomials listed in Table 14.1.

	recursion formula
$P_n(x)$	$(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0$
$T_n(x)$	$T_{n+1}(x) - 2xT_n(x) + T_{n-1}(x) = 0$
$L_n^{(\alpha)}(x)$	$(n+1)L_{n+1}^{(\alpha)}(x) + (x - 2n - 1 - \alpha)L_n^{(\alpha)}(x) + (n+\alpha)L_{n-1}^{(\alpha)}(x) = 0$
$H_n(x)$	$H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0$
$He_n(x)$	$He_{n+1}(x) - xHe_n(x) + nHe_{n-1}(x) = 0$

Table 14.3

The generating functions of the orthogonal polynomials listed in Table 14.1.

	generating function
$P_n(x)$	$(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n \quad \lvert t\rvert < 1$
$T_n(x)$	$\dfrac{1-xt}{1-2xt+t^2} = \sum_{n=0}^{\infty} T_n(x)t^n \quad \lvert t\rvert < 1$
$L_n^{(\alpha)}(x)$	$\dfrac{e^{-xt/(1-t)}}{(1-t)^{\alpha+1}} = \sum_{n=0}^{\infty} L_n^{(\alpha)}(x)t^n \quad \lvert t\rvert < 1$
$H_n(x)$	$e^{2xt-t^2} = \sum_{n=0}^{\infty} \dfrac{1}{n!}H_n(x)t^n$

Table 14.5

The differential equation for the orthogonal polynomials listed in Table 14.1.

	differential equation
$P_n(x)$	$(1-x^2)y''(x) - 2xy'(x) + n(n+1)y(x) = 0$
$T_n(x)$	$(1-x^2)y''(x) - xy'(x) + n^2y(x) = 0$
$L_n^{(\alpha)}(x)$	$xy''(x) + (\alpha+1-x)y'(x) + ny(x) = 0$
$H_n(x)$	$y''(x) - 2xy'(x) + 2ny(x) = 0$
$He_n(x)$	$y''(x) - xy'(x) + ny(x) = 0$